RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2017

THIRD YEAR (BATCH 2015-18)

PHYSICS (Honours)

Paper : VI

Date : 21/12/2017

Time : 11.00 am – 1.00 pm

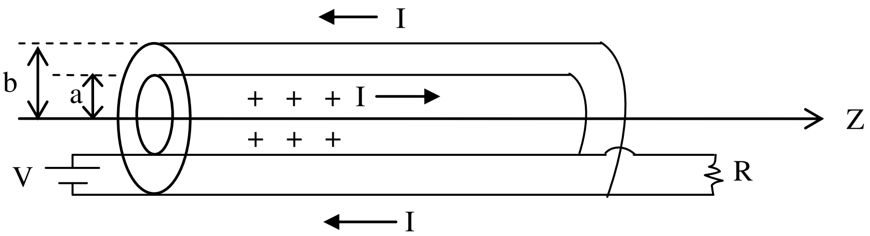
Full Marks : 50

Answer **any five** questions: (5 × 10)

1. a) Write down Maxwell's field equations in free space and interpret each equation in physical terms. 3
   b) Show that these equations are consistent with the law of conservation of charge. 1
   c) Show that in a source-free region of space, the electric and magnetic field vectors satisfy identical wave equations. 2
   d) Obtain a sinusoidal plane-wave solution of the wave equation for an electromagnetic wave propagating in the +z direction. Show that these waves are necessarily transverse. 4

2. a) Starting from Maxwell's field equations in free space, obtain the corresponding field equations for any arbitrary but linear, homogeneous and isotropic non-conducting medium. Are these equations complete in themselves? Explain. 4
   b) Deduce Poynting's theorem in integral form, and discuss its physical significance. What conservation law does it express? 6

3. a) Show that if $\vec{E}$ and $\vec{H}$ are complex, harmonic solutions of Maxwell's equations, then the time-averaged Poynting vector $\langle\vec{S}\rangle$ can be expressed as $\langle\vec{S}\rangle = \frac{1}{2}\mathrm{Re}\left(\vec{E}\times\vec{H}^*\right)$. 4
   b) If the electromagnetic field energy density in a plane wave is $u_{em} = \frac{1}{2}\vec{E}\cdot\vec{D} + \frac{1}{2}\vec{H}\cdot\vec{B}$, show that $\langle\vec{S}\rangle = v\langle u_{em}\rangle\hat{u}$, where v is the wave speed and $\hat{u}$ is the direction of propagation of the wave. (Symbols have usual meaning) 2
   c)



   A long coaxial cable, of length l, consists of an inner conductor (radius a) and an outer conductor (radius b). It is connected to a battery at one end and a resistor at the other end. The inner conductor carries a uniform charge per unit length λ, and a steady current I to the right. The outer conductor has the opposite charge and current. What is the power transported? 4

4. a) Write down Maxwell's field equations for the Electromagnetic (EM) field in a linear isotropic conductor having conductivity σ_1 permittivity ϵ_1, permeability μ_0. Show that such a conductor cannot sustain any volume charge density. 3
   b) Obtain the wave equations for the EM field vectors in the conductor. Hence show that if the fields vary harmonically in time with frequency ω, the propagation constant K is necessarily complex. 2

c) i) Show that the skin depth in a poor conductor $(\sigma \ll \omega \in)$ is $\dfrac{2}{\sigma}\sqrt{\dfrac{\in}{\mu}}$. 2

ii) Show that the skin depth in a good conductor $(\sigma \gg \omega \in)$ is $\dfrac{\lambda}{2\pi}$. For a typical metal $\left(\sigma \approx 10^{7}\,\Omega^{-1}m^{-1}\right)$ compute the skin depth in the visible range $\left(\omega \approx 10^{15}/\mathrm{sec}\right)$. Assume $\in \approx \in_0$ and $\mu \approx \mu_0$. 2

iii) Show that in a good conductor the magnetic field lags the electric field by $\dfrac{\pi}{4}$. 1

5. a) Express Maxwell's equations in integral form and write down the boundary conditions for electric and magnetic fields at a boundary between two linear media. (Assume that there is no free charge or free current at the interface). 1+3

b) A plane monochromatic electromagnetic (EM) wave of angular frequency ω is incident normally on the plane interface of two distinct dielectrics, having refractive indices n_1 and n_2, respectively. Choose the z direction as the normal to the interface, which is the $x - y$ plane. 6

i) Is there a unique plane of incidence?

ii) Argue that the reflected and transmitted waves are directed along the normal.

iii) Show that the amplitude reflection and amplitude transmission coefficients r and t are given by, $r = \dfrac{n_1 - n_2}{n_1 + n_2}$, $t = \dfrac{2n_1}{n_1 + n_2}$; ($\mu_1 = \mu_2 = \mu_0$; μ_1 & μ_2 are permeabilities of two distinct dielectric).

iv) If the permittivities of two distinct dielectric are $\in_1 = 6.25 \in_0$, $\in_2 = 15.21 \in_0$, then find t and r and the phase difference between incident and reflected waves.

OR

a) Deduce Fresnel's equations for the amplitude reflection and transmission coefficients when the electric field vectors are all in the plane of incidence. (Assume a plane interface of two dielectrics of refractive indices n_1 and n_2 respectively). 4

b) A plane-polarized wave of amplitude E_0 in vacuum is incident normally on a dielectric of refractive index n. 3

i) Find the electric and magnetic field amplitudes in the dielectric.

ii) If the incident wave has a power density 1 KW/m^2, calculate the average Poynting vector in the dielectric, if $n = 1.5$.

c) Define Brewster angle θ_p, and show that if an EM wave with arbitrary state of polarization be incident on the interface of the dielectrics at an angle θ_p, the reflected wave is p–polarized. Can this Brewster effect occur on reflection from a less-dense medium? 3

6. a) Explain briefly what you mean by 'dispersion', and a 'dispersive' medium, in the context of propagation of an electromagnetic wave. 1

b) A polychromatic electromagnetic (EM) wave propagating in a dielectric medium is both dispersed and absorbed. Modelling the dielectric as composed of molecules having only are resonant frequency ω_0, obtain expressions for the refractive index $n(\omega)$ and the absorption coefficient $K(\omega)$, as a function of the wave frequency ω, in the limit of low density. 4

c) i) Plot $n(\omega)$ and $K(\omega)$ as a function of ω.

ii) In the $n(\omega)$ plot, identify the regions of normal and anomalous dispersion.

iii) Show that the region of anomalous dispersion coincides with the region of maximum absorption. (assume $\gamma \ll \omega_0$). [γ =damping coefficient] 1+1+3

7. a) What is Rayleigh's scattering? In the absence of atmosphere, the sky would appear black in day time. Explain. 2+2

b) What is double refraction? How Huygens' theory describe the phenomenon of double refraction? 1+5

8. a) What is a quarter wave plate? Discuss how it can be used to produce circularly and elliptically polarized light. 1+3

b) A left circularly polarised beam $\left(\lambda_0 = 5893 \text{ Å}\right)$ is incident on a quartz crystal (with its optic axis cut parallel to the surface) of thickness 0.025 mm. Determine the state of polarisation of emergent beam. Assume $n_0 = 1.54425$, $n_e = 1.55336$ are refraction indices. 3

c) Discuss Brewster's law regarding polarization by reflection. 3

——— × ———